

ABSTRACT

This paper deals with the effect of heat generation (absorption) of MHD forced convection flow of incompressible, electrically conducting viscous fluid over a moving cylindrical rod with thermal radiation. The system of coupled partial differential equations governing the flow and heat transfer is solved using an efficient implicit finite - difference scheme along with quasilinearization technique. Numerical computations are performed for air ($Pr = 0.7$) and displayed graphically to illustrate the influence of relevant physical parameters on local skin friction and heat transfer coefficients and, also on, velocity and temperature fields. It is observed that the heat generation decreases the heat transfer whereas heat absorption does the opposite. Further, heat generation reduces the thickness of thermal boundary layer, whereas it increases due to heat absorption. It is also observed that the heat transfer coefficient and temperature fields are strongly affected due to thermal radiation.

KEYWORDS: Magnetic field, Heat generation (absorption), Thermal radiation, temperature, Heat transfer coefficient.

INTRODUCTION

Boundary layer flows over a moving or stretching surface are of great importance in view of their relevance to a wide variety of technical applications, especially in the manufacture of fibers in glass and polymer industries. Let us consider the steady laminar flow induced by the motion of a cylindrical rod issuing from an orifice into a fluid at rest. The boundary layer behavior on moving surface in a quiescent fluid was first considered by Sakiadis [1]. Jaffe and Okamura [2] studied the transfer curvature effect on the incompressible laminar boundary layer longitudinal flow over a cylinder. The nature of boundary layer flow on a cylinder moving in a fluid at rest has been investigated by Crane [3]. Kuiken[4] discussed the cooling of heat resistant cylinder moving through a fluid. The laminar boundary layer flow on a moving cylindrical rod has been examined by zachara [5]. Also, Brendan and David [6] applied Hausen integral method to find the rate of heat loss of the fiber during the manufacture of polymer fibers. Heat transfer over stretching sheet in a stagnation point with magnetic field and chemically reactive species are examined by Kumari and Nath [7] and Thakar etc.all[8] respectively. MHD boundary layer flow due to continuous moving flat plate has been discussed by Chiam [9]. The laminar boundary layer flow on a moving cylindrical rod has been studied by Jayakumar and Eswara[10] with an applied magnetic field. Recently, MHD laminar boundary layer flow and heat transfer has been investigated in detail along with viscous dissipation using finite difference method by Jayakumar and Eswara[11].

The objective of the present paper is to analyse the effect of heat generation (absorption) and thermal radiation on laminar boundary layer flow over a moving cylindrical rod with an applied magnetic field.

ANALYSIS

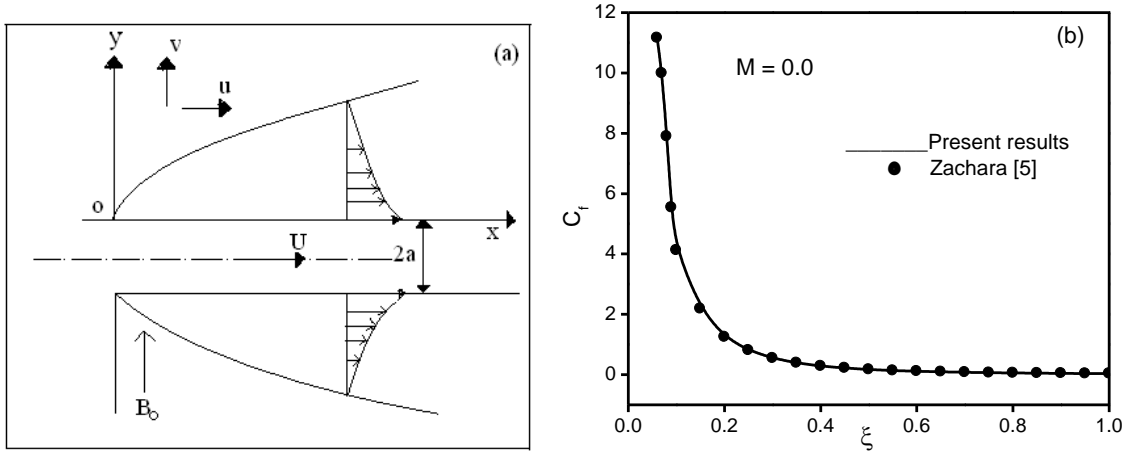


Fig.1 (a) Flow model and co – ordinate system
(b) Comparison of skin friction coefficient (C_f) with that of Zachara [5]

Let us consider the steady, axisymmetric laminar boundary layer of an incompressible fluid with an applied magnetic field on a continuous cylinder moving from an orifice in an axial direction at constant velocity through a fluid at rest. The radius of the cylinder and its velocity are denoted by “a” and “U”. A magnetic field B_0 fixed relative to the fluid is applied in y – direction. (See Fig.1(a)). It is assumed that magnetic Reynolds number is small so that the induced magnetic field can be neglected. The flow is considered in frames of the boundary layer co – ordinate system in which the x – axis, parallel to the axis of symmetry, is posed along the solid surface and the y – axis is normal to it. The origin of the co – ordinate system is put in the plane of the orifice. Under the aforementioned assumptions, the equations governing the above nonsimilar flow are [5]:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu}{r} \frac{\partial}{\partial y} \left(r \frac{\partial u}{\partial y} \right) - \frac{\sigma B_0^2}{\rho} (u - U) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{q_0}{\rho c_p} (T - T_\infty) \tag{3}$$

where $r(y) = a + y$

The boundary conditions are

$$u(x,0) = U; \quad v(x,0) = 0; \quad u(x,\infty) = 0; \quad T(x,0) = T_w; \quad T(x,\infty) = T_\infty \tag{4}$$

Here, the radiative heat flux q_r , under Roseland approximation, has the form

$$q_r = - \frac{4\sigma^*}{3k} \frac{\partial T^4}{\partial z} \tag{5}$$

Expanding T^4 in a Taylor series about T_∞ and neglecting higher orders yields:

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

Substituting (5) and (6) into (3) gives,

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha (1 + N_R) \frac{\partial^2 T}{\partial y^2} + \frac{q_0}{\rho c_p} (T - T_\infty) \tag{7}$$

The Eq.(2) and (7) are transformed into nondimensional form using the combination of the Mangler and Falkner – Skan transformations [7]:

$$\eta = \left(\frac{U}{\nu x}\right)^{1/2} y \left(1 + \frac{y}{2a}\right); \quad \xi = \left(\frac{4\nu x}{Ua^2}\right)^{1/2}; \quad \psi(x, y) = a(U\nu x)^{1/2} f(\xi, \eta);$$

$$T = T_\infty + (T_w - T_\infty) G(\xi, \eta) \quad (8)$$

which satisfies the continuity Eq.(1). Consequently, the momentum Eq.(2) and energy Eqn.(7) becomes

$$(1 + \xi \eta) f'''' + \xi f'' + \frac{ff''}{2} - M(f' - 1) = \frac{\xi}{2} (f' f'_\xi - f'' f_\xi) \quad (9)$$

$$(1 + N_R) \left[(1 + \xi \eta) G'' + \frac{\xi G'}{2} \right] + \frac{f G' \text{Pr}}{2} + \frac{QG \text{Pr}}{2} = \frac{\xi \text{Pr}}{2} (f' G_\xi - G' f_\xi) \quad (10)$$

where

$$u = Uf'; \quad v = -\frac{a}{r} \left(\frac{\nu U}{4x}\right)^{1/2} \{f - \eta f' + \xi f_\xi\}; \quad M = \frac{x\sigma B_0^2}{\rho U};$$

$$\text{Pr} = \frac{\nu}{\alpha}; \quad Q = \frac{xq_0}{U\rho c_p}; \quad N_R = \frac{8\sigma^* T_\infty^3}{3k^2} \quad (11)$$

The transformed boundary conditions are:

$$f(\xi, 0) = 0; \quad f'(\xi, 0) = 1; \quad f'(\xi, \infty) = 0$$

$$G(\xi, 0) = 1; \quad G(\xi, \infty) = 0 \quad (12)$$

for $\xi \geq 0$.

The skin friction and heat transfer coefficients are defined as

$$C_f = -\frac{a\tau(x, a)}{\mu U} = -\frac{2}{\xi} f''(0); \quad Nu = -\frac{a\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} = -\frac{2}{\xi} G'(0) \quad (13)$$

Here u and v are the velocity components along x - and y - directions respectively; ξ and η are transformed co-ordinates; ψ and f are the dimensional and dimensionless stream functions, respectively; M is the nondimensional magnetic parameter; The subscript ξ denote partial derivative with respect to ξ and prime ($'$) denote derivatives with respect to η .

It is worth mentioning here that when $M = 0.0$, Eq.(9) reduces to

$$(1 + \xi \eta) f'''' + \xi f'' + \frac{ff''}{2} = \frac{\xi}{2} (f' f'_\xi - f'' f_\xi) \quad (14)$$

which is exactly same as that of A.Zachara [5]. It is also noted here that when $Q = 0.0$ and $NR = 0.0$ the Eqns. (9) and (10) reduces to

$$(1 + \xi \eta) f'''' + \xi f'' + \frac{ff''}{2} - M(f' - 1) = \frac{\xi}{2} (f' f'_\xi - f'' f_\xi) \quad (15)$$

$$(1 + \xi \eta) G'' + \frac{\xi G'}{2} + \frac{f G' \text{Pr}}{2} = \frac{\xi \text{Pr}}{2} (f' G_\xi - G' f_\xi) \quad (16)$$

which are exactly same as those of Jayakumar and Eswara [11] in the presence of magnetic field.

RESULTS AND DISCUSSIONS

The partial differential Eqs.(9) and (10) along with boundary condition (12) has been solved numerically employing finite difference method in combination with quasilinearization technique. Since the method is described in great detail in [12 and 13], its description is omitted here, for the sake of brevity.

In order to assess the accuracy of our method of solution, we have compared our skin friction coefficient (C_f) for $M = 0.0$ (i.e., without magnetic field) with that of Zachara [5] by solving the Eq.(14). [See Fig.1(b)]. Also, the velocity and temperature profiles are computed by solving Eqns. (15) and (16) without thermal radiation and heat

generation (absorption) compared with Jayakumar and Eswara [11] [Fig.2(a) and 2(b)]. Our results are found to be in excellent agreement with the above study.

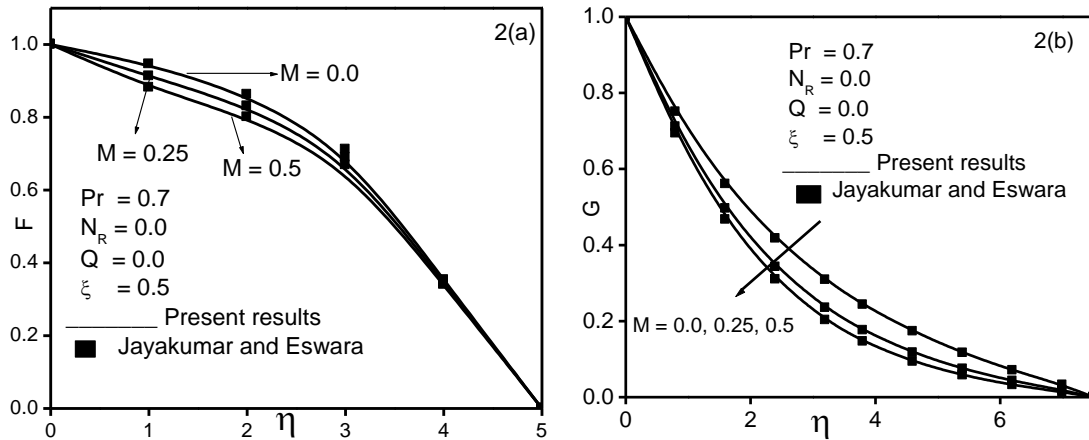


Fig.2 Comparison of (a) Velocity (b) temperature profiles with Jayakumar and Eswara[11]

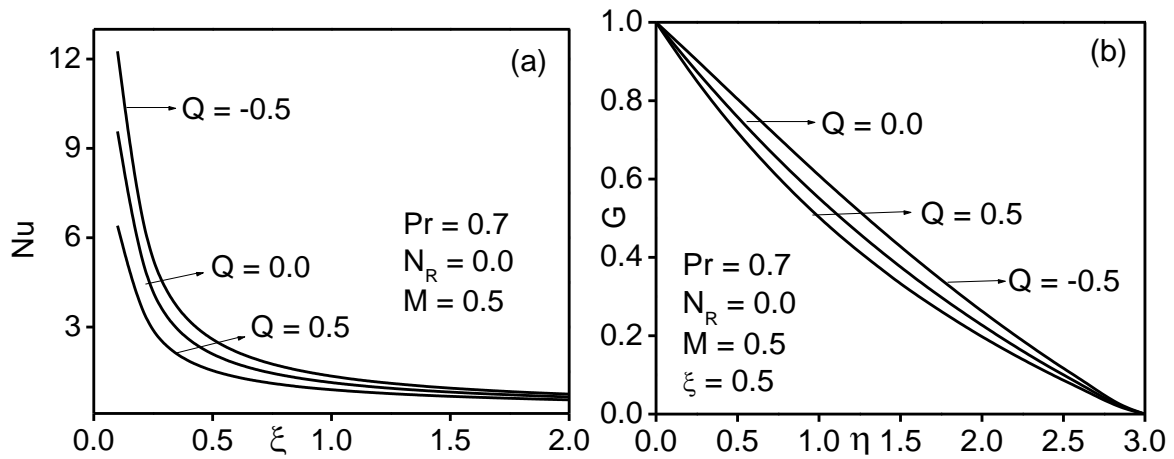


Fig.3 Effect of heat generation (absorption) on (a) heat transfer (Nu) (b) temperature profile (G)

Fig.3. depicts the influence of heat generation ($Q > 0$) and heat absorption ($Q < 0$) on heat transfer (Nu) and on temperature (G) along with mild magnetic field ($M = 0.5$). From the graph it is clear that the heat generation reduces the heat transfer (Nu) whereas heat absorption enhances Nu . [Fig.3(a)] Indeed, the percentage of decrease of heat transfer is 5% and the percentage of increase of Nu is 6% at $\xi = 1.0$. It is noted from fig2 (b) that the thickness of thermal boundary layer decreases due to heat generation and it increases for heat absorption.

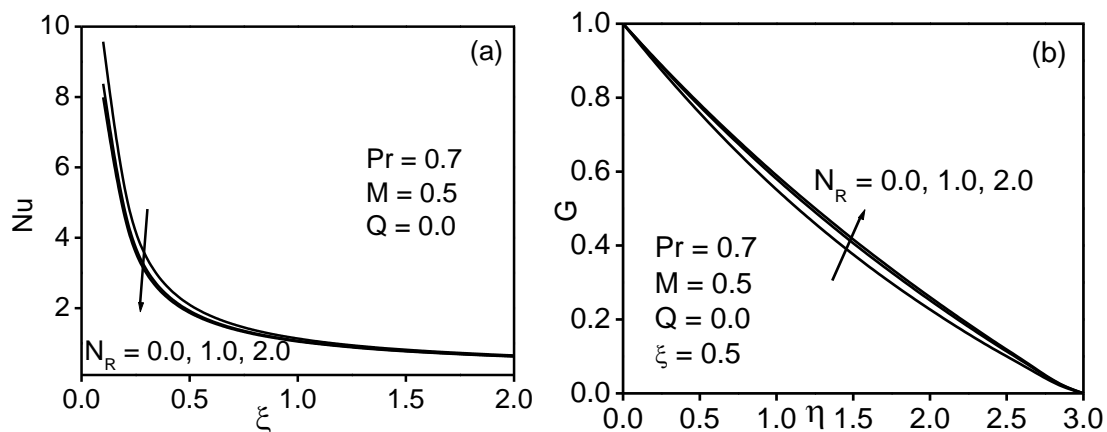


Fig.4 Effect of thermal radiation on (a) heat transfer (Nu) (b) temperature (G)

The effect of thermal radiation (N_R) on heat transfer (Nu) and the corresponding temperature profile in presence of magnetic field are displayed in Fig.4. The thermal radiation decreases Nu [Fig.4(a)], while its effect is just opposite on temperature profile (G) [Fig.4(b)] for all stream wise locations ξ . The percentage of decrease of Nu from $N_R = 0.0$ to $N_R = 2.0$ is about 8.25 % at an arbitrary value of ξ ($\xi = 1.0$). The temperature boundary layer increase about 3.2% in the range $0.0 \leq N_R \leq 2.0$ near $\eta = 2.0$.

CONCLUSIONS

The effect of heat generation (absorption) and thermal radiation has been investigated on forced convection laminar boundary layer flow on a moving cylindrical rod with an applied magnetic field. It is observed that the heat generation decreases the heat transfer whereas heat absorption increases it also it is noted that the heat generation reduces the temperature whereas heat absorption enhances the temperature. Further, the heat transfer coefficient decreases and the thickness of thermal boundary layer increases with the increase of thermal radiation parameter.

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